



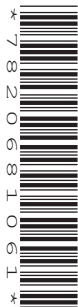
Oxford Cambridge and RSA

**Thursday 7 October 2021 – Afternoon**

**A Level Further Mathematics A**

**Y541/01 Pure Core 2**

**Time allowed: 1 hour 30 minutes**



**You must have:**

- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

**INSTRUCTIONS**

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . When a numerical value is needed use  $g = 9.8$  unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

**INFORMATION**

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [ ].
- This document has **8** pages.

**ADVICE**

- Read each question carefully before you start your answer.

Answer **all** the questions.

- 1** Two matrices, **A** and **B**, are given by  $\mathbf{A} = \begin{pmatrix} 1 & -2 & -1 \\ 2 & -3 & 1 \\ a & 1 & 1 \end{pmatrix}$  and  $\mathbf{B} = \begin{pmatrix} -6 & 3 & -4 \\ -1 & 6 & -4 \\ 8 & -8 & -1 \end{pmatrix}$  where  $a$  is a constant.
- Find the value of  $a$  for which  $\mathbf{AB} = \mathbf{BA}$ . **[3]**

**2 In this question you must show detailed reasoning.**

The complex numbers  $z_1$  and  $z_2$  are given by  $z_1 = 3 - 7i$  and  $z_2 = 2 + 4i$ .

- (a)** Express each of the following as exact numbers in the form  $a + bi$ .

**(i)**  $3z_1 + 4z_2$  **[1]**

**(ii)**  $z_1 z_2$  **[2]**

**(iii)**  $\frac{z_1}{z_2}$  **[2]**

- (b)** Write  $z_1$  in modulus-argument form giving the modulus in exact form and the argument correct to **3** significant figures. **[3]**

3 The line  $l_1$  has equation  $\mathbf{r} = \begin{pmatrix} 1 \\ -3 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix}$ .

The plane  $\Pi$  has equation  $\mathbf{r} \cdot \begin{pmatrix} 2 \\ -5 \\ -3 \end{pmatrix} = 4$ .

(a) Find the position vector of the point of intersection of  $l_1$  and  $\Pi$ . [3]

(b) Find the acute angle between  $l_1$  and  $\Pi$ . [3]

$A$  is the point on  $l_1$  where  $\lambda = 1$ .

$l_2$  is the line with the following properties.

- $l_2$  passes through  $A$
- $l_2$  is perpendicular to  $l_1$
- $l_2$  is parallel to  $\Pi$

(c) Find, in vector form, the equation of  $l_2$ . [3]

4 In this question you must show detailed reasoning.

Determine the value of  $\sum_{r=1}^{100} (2r+3)^2$ . [3]

5 In this question you must show detailed reasoning.

(a) Using the definition of  $\cosh x$  in terms of exponentials, show that  $\cosh 2x \equiv 2\cosh^2 x - 1$ . [2]

(b) Solve the equation  $\cosh 2x = 3\cosh x + 1$ , giving all your answers in exact logarithmic form. [6]

**6 In this question you must show detailed reasoning.**

The matrix **A** is given by  $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ .

(a) Define the transformation represented by **A**. [1]

(b) Show that the area of any object shape is invariant under the transformation represented by **A**. [1]

The matrix **B** is given by  $\mathbf{B} = \begin{pmatrix} 7 & 2 \\ 21 & 7 \end{pmatrix}$ . You are given that **B** represents the transformation which is the result of applying the following **three** transformations in the given order.

- A shear which leaves the  $y$ -axis invariant and which transforms the point (1, 1) to the point (1, 4).
- The transformation represented by **A**.
- A stretch of scale factor  $p$  which leaves the  $x$ -axis invariant.

(c) Determine the value of  $p$ . [4]

**7 In this question you must show detailed reasoning.**

(a) Find the values of  $A$ ,  $B$  and  $C$  for which  $\frac{x^3 + x^2 + 9x - 1}{x^3 + x^2 + 4x + 4} \equiv A + \frac{Bx + C}{x^3 + x^2 + 4x + 4}$ . [1]

(b) Hence express  $\frac{x^3 + x^2 + 9x - 1}{x^3 + x^2 + 4x + 4}$  using partial fractions. [5]

(c) Using your answer to part (b), determine  $\int_0^2 \frac{x^3 + x^2 + 9x - 1}{x^3 + x^2 + 4x + 4} dx$  expressing your answer in the form  $a + \ln b + c\pi$  where  $a$  is an integer, and  $b$  and  $c$  are both rational. [4]

- 8 A particle  $P$  of mass 2 kg can only move along the straight line segment  $OA$ , where  $OA$  is on a rough horizontal surface. The particle is initially at rest at  $O$  and the distance  $OA$  is 0.9 m.

When the time is  $t$  seconds the displacement of  $P$  from  $O$  is  $x$  m and the velocity of  $P$  is  $v$  ms<sup>-1</sup>.  $P$  is subject to a force of magnitude  $4e^{-2t}$  N in the direction of  $A$  for any  $t \geq 0$ . The resistance to the motion of  $P$  is modelled as being proportional to  $v$ .

At the instant when  $t = \ln 2$ ,  $v = 0.5$  and the resultant force on  $P$  is 0 N.

- (a) Show that, according to the model,  $\frac{dv}{dt} + v = 2e^{-2t}$ . [3]
- (b) Find an expression for  $v$  in terms of  $t$  for  $t \geq 0$ . [5]
- (c) By considering the behaviour of  $v$  as  $t$  becomes large explain why, according to the model,  $P$ 's speed must reach a maximum value for some  $t > 0$ . [2]
- (d) Determine the maximum speed considered in part (c). [2]
- (e) Determine the greatest value of  $t$  for which the model is valid. [4]

- 9 The matrix  $\mathbf{A}$  is given by  $\mathbf{A} = \begin{pmatrix} 2 & 3 \\ 0 & 2 \end{pmatrix}$ .

- (a) By considering  $\mathbf{A}$ ,  $\mathbf{A}^2$ ,  $\mathbf{A}^3$  and  $\mathbf{A}^4$  make a conjecture about the form of the matrix  $\mathbf{A}^n$  in terms of  $n$  for  $n \geq 1$ . [2]
- (b) Use induction to prove the conjecture made in part (a). [4]

**10 In this question you must show detailed reasoning.**

- (a) By using an appropriate Maclaurin series prove that if  $x > 0$  then  $e^x > 1 + x$ . [2]
- (b) Hence, by using a suitable substitution, deduce that  $e^t > et$  for  $t > 1$ . [1]
- (c) Using the inequality in part (b), and by making a suitable choice for  $t$ , determine which is greater,  $e^\pi$  or  $\pi^e$ . [3]

**END OF QUESTION PAPER**

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